Aeroelasticity of Laminated Fiber-Reinforced Doubly Curved Shallow Shells

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I. Introduction

LAMINATED fiber-reinforced composite materials are being utilized increasingly in the design of exposed-skin construction of supersonic and re-entry vehicles. In general, such panels have a greater strength-to-weight ratio than the conventional isotropic panels and, thus, provide considerable weight savings. However, their use introduces several complication factors that are not present in the conventional isotropic panels. Such complications are mainly due to the fiber orientation, which introduces a twisting–bending coupling, and the number of layers and their stacking sequence, which introduce a material stretching–bending coupling. A further complication is introduced by the geometric stretching–bending coupling due to the shell curvature. All of these factors interact in a complicated manner on the free vibration frequency spectrum of the shells and, therefore, affect their borderline of dynamic stability. Aeroelasticity of plates and shells constitutes an important aspect in the design of high-speed vehicles. Two monographs[1,2] completely devoted to the subject were published in the 1970s, and several reviews and survey papers[3–8] related to the subject are available in the scientific literature. Since the earlier works on panel flutter, the complications introduced in the design due to the use of composite materials were addressed by several investigators, for example, Refs. 9–12. These pioneering works mainly concentrated on flat orthotropic panels and used the Rayleigh–Ritz and Galerkin methods for the problem solution. With the advent of high-speed computation devices and the efficient use of the finite element method in structural dynamic stability problems, much research on the aeroelasticity of fiber-reinforced composite material panels using the finite element method was published, for example, Refs. 13–17. In all of these studies, the total potential energy functional was used for the finite element formulation of the problem. Alternatively, the problem can be formulated using a two-field variable modified functional, with the transverse displacement w and Airy stress function F as the field variables of the problem. In spite of the simplification it introduces, the two-field variable principle is rarely used in finite element applications. The main reason is the complications introduced for the application of the boundary conditions on the Airy stress function. In Ref. 18, starting from Reissner's variational equation for free vibration of thin isotropic cylindrical curved plates, the Euler–Lagrange equations governing the problem and the boundary conditions were obtained. It was shown that the boundary conditions on F are as simple and direct to apply as on w (Ref. 18). The formulation was then extended to the buckling analysis of cylindrically curved plates,[19,20] to the supersonic flutter of cylindrically curved isotropic panels,[21] and to the problem of stability of cantilever cylindrically curved isotropic panels subjected to nonconservative follower forces.[22] The purpose of the present work is to present a two-field variable variational formulation, with w and F as the field variables, for the analysis of fiber-reinforced doubly curved shallow shells subjected to nonconservative aerodynamic loads. It is shown that the functional presented has no explicit material bending–extensional coupling terms. These effects appear only in the equivalent material bending stiffness constitutive constants. The solution of the problem is made using a C^1 continuity finite element method.

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II. Problem Formulation

The variational equation of laminated fiber-reinforced composite material, doubly curved shallow shells, considering the effect of the work done by external incremental nonstationary airloads applied to the upper surface, can be written as

$$\begin{aligned}\delta(\Pi^*) = \delta\int_{t_1}^{t_2}\Bigg[&\frac{1}{2}\int_A \rho h w_{,t}^2\,\mathrm{d}A - \int_A w\left\{\frac{F_{,xx}}{R_x}+\frac{F_{,yy}}{R_y}\right\}\mathrm{d}A\\
&-\frac{1}{2}\int_A\Big[D_{11}^* w_{,xx}^2 + D_{22}^* w_{,yy}^2 + 2D_{12}^* w_{,xx}w_{,yy} + 4D_{33}^* w_{,xy}^2\\
&+4D_{13}^* w_{,xx}w_{,xy} + 4D_{23}^* w_{,yy}w_{,xy}\Big]\mathrm{d}A + \frac{1}{2}\int_A\Big[A_{22}^* F_{,xx}^2\\
&+A_{11}^* F_{,yy}^2 + 2A_{12}^* F_{,xx}F_{,yy} + 4A_{33}^* F_{,xy}^2 - 2A_{23}^* F_{,xx}F_{,xy}\\
&-2A_{13}^* F_{,yy}F_{,xy}\Big]\mathrm{d}A + \int_A w\Delta p\,\mathrm{d}A\Bigg]\mathrm{d}t = 0\end{aligned} \tag{1}$$

where ρ is the material mass density, h is the shell total thickness, and R_x and R_y are the shell radii of curvature in the x and y directions, respectively. The material constitutive constants are obtained from the relation

$$\begin{Bmatrix}\{\varepsilon^0\}\\ \{M\}\end{Bmatrix} = \begin{bmatrix}[A^*] & [-A^{-1}B]\\ [BA^{-1}] & [D^*]\end{bmatrix}\begin{Bmatrix}\{N\}\\ \{\kappa\}\end{Bmatrix} \tag{2}$$

where $\{M\}$ and $\{N\}$ are the stress and moment resultants, $\{\varepsilon^0\}$ and $\{\kappa\}$ are the middle surface strain and curvature vectors, and $[A^*] = [A]^{-1}$ and $[D^*] = [D] - [B][A]^{-1}[B]$. The global constitutive matrices $[A]$, $[B]$, and $[D]$ of the laminated fiber-reinforced composite material are obtained from the laminae's properties Q_{ij}^* in the reference coordinates and can be written as[23,24]

$$A_{ij} = \sum_{k=1}^{n} Q_{ijk}^*(h_k - h_{k-1}), \qquad B_{ij} = \frac{1}{2}\sum_{k=1}^{n} Q_{ijk}^*\left(h_k^2 - h_{k-1}^2\right)$$
$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} Q_{ijk}^*\left(h_k^3 - h_{k-1}^3\right) \tag{3}$$

where h_k is the vectorial distance from the middle surface to the upper surface of lamina k and n is the total number of laminae. Notice the simplicity of the present formulation, Eq. (1), where we do not have explicit material bending–in-plane coupling terms, their effects appearing only in the equivalent constitutive elements D_{ij}^*. Using the quasistatic aerodynamic theory, the relationship between the incremental nonstationary aerodynamic pressure Δp and the transverse displacement w, in Eq. (1), can be written as

$$\Delta p = -\frac{2Q}{(M^2-1)^{\frac{1}{2}}}\frac{\partial w}{\partial x} \tag{4}$$

where $Q = \rho V^2/2$ is the dynamic pressure and M and V are the freestream Mach number and the velocity, respectively. Now a finite element solution for the problem at hand can be performed, using rectangular elements preserving C^1 continuity, based on the functional given in Eq. (1), using first-order Hermitian polynomials. The element matrices and the boundary conditions are obtained in the same manner as reported in Refs. 18–22 and 25, and the finite element matrix equations for the whole structure can be written as

$$[K_{ww}]\{w\} + [K_{wF}]\{F\} + [M]\{\ddot{w}\} + \lambda[A]\{w\} = \{0\} \tag{5a}$$

$$[K_{Fw}]\{w\} + [K_{FF}]\{F\} = \{0\} \tag{5b}$$

where λ is the dynamic pressure parameter, equal to $2Q/(M^2-1)^{1/2}$. The degrees of freedom $\{F\}$ can be eliminated using the compatibility equation (5b) to obtain

$$[K_{\mathrm{eq}}]\{w\} + \lambda[A]\{w\} + [M]\{\ddot{w}\} = \{0\} \tag{6}$$

where $[K_{eq}] = [K_{ww}] - [K_{wF}][K_{FF}]^{-1}[K_{Fw}]$. An examination of Eq. (6) reveals that the computational effort required for the solution of the stability problem is equivalent to that of a flat-plate problem when the present formulation is used.

III. Numerical Results and Discussion

The problem of vibration and flutter of doubly curved laminated fiber-reinforced composite material shells presents several complication factors. The fiber orientation, the number of layers, and their stacking sequence introduce extension–bending, twisting–bending, and extension–shear couplings. The shell geometric curvature presents a further extension–bending interaction effect. The transverse and in-plane boundary conditions affect the natural vibration frequency spectrum and, therefore, the borderline of the flutter stability of the shell. To study the effect and the trend of these parameters on the stability of the shell in a systematic and organized manner, several examples have been analyzed. These examples address one or more parameters at a time to determine their effect on the borderline of the flutter stability of the shell.

The first examples analyzed are for flutter solutions of flat single-layer orthotropic rectangular panels. The flutter problem was addressed in earlier works on orthotropic panel flutter[9–12] and recent finite element method solutions.[14,16,17] For these plates the only coupling present is that due to twisting–bending when the fiber orientation angle is not aligned with the plate reference axis. The results obtained using the present formulation for a 4×4 mesh coincide with previous finite element solutions[14,16] for a square planform, and the maximum dynamic pressure parameter is attained when the orthotropicity angle is zero. For plates with aspect ratio other than one, the present finite element solution duplicates the earlier analytical solutions,[9–12] where it was observed that a local maximum for the dynamic pressure parameter is reached at an orthotropicity angle between 0 and 90 deg (in the vicinity of 30 deg for $a/b = 3$).

In the next series of calculations, the effects of the number of layers and of their stacking sequence on the borderline of the flutter stability of laminated fiber-reinforced composite panels were examined. Examples of flat plates with symmetrically disposed layers, i.e., no material bending–stretching coupling, and asymmetrically stacked layers, i.e., with material bending–stretching coupling, were analyzed. The results obtained using the present formulation showed favorable agreement with the previous analysis,[13,15,16] and it was observed that the material bending–stretching coupling has a big influence on the flutter boundary and is destabilizing. The flutter dynamic pressure parameter for the symmetric stacking arrangement is more than twice the value for the asymmetric arrangement. In the sequel the effect of the bending–stretching coupling, due to the geometric curvature, on the borderline of flutter stability will be studied in detail. First, a series of free vibration analyses of cross-ply [0/90] shallow spherically and cylindrically curved shells was performed. The results obtained using the present formulation agree favorably with the finite element solution of Ref. 26 and the analytical solution of Ref. 27.

The final series of calculations presented are flutter solutions of doubly curved laminated fiber-reinforced composite material shallow shells. The material properties common to all of these calculations are $E_1 = 21 \times 10^6$ psi, $E_2 = 0.84 \times 10^6$ psi, $G_{12} = 0.42 \times 10^6$ psi, $\upsilon_{12} = 0.25$, and four layers [0/90/90/0]. The shells analyzed have a planform of dimensions 100×100 in. and a thickness of 1 in. The calculations were performed for cylindrical shells, paraboloidal shells, and spherical shells. The boundary conditions considered are freely supported boundary conditions on the four edges and clamped boundary conditions on all edges. All of the analyses were performed using a finite element mesh of 4×4 elements. The results of the analyses are summarized in Figs. 1 and 2. In these calculations R_1 was considered as the radius of curvature in the cross-stream direction and is common for all of the types of shells analyzed. In the streamwise direction R_2 is infinity for the cylindrical shells, $R_2 = R_1$ for the spherical shells, and R_2 was taken as $2R_1$ for the paraboloidal shells. From the results of the analyses, it can be concluded that as was expected, the clamped boundary conditions present higher critical dynamic pressure values as compared to the freely supported boundary conditions. Furthermore, the effect of the streamwise curvature is destabilizing, i.e., for the same cross-stream curvature the cylindrical shell is more stable than the paraboloidal shell and this is more stable than the spherical shell. The effect of cross-stream curvature is similar to the case of isotropic shallow shells analyzed in Refs. 20 and 21. For very small curvature, the critical flutter modes are the first modes and λ^*_{cr} is practically the same as for a flat panel. With the increase in curvature, higher modes coalesce first, and the coalescence is characterized by the decrease or increase in the critical dynamic pressure parameter. In the region of the flat plate behavior, the curvature effect is stabilizing. With the increase in curvature the shell passes through a transition region, characterized by the successive waves of successive-higher-mode coalescence. After this transition region the panel behaves as a deep shell and λ^*_{cr} is for an elevated number of waves in the cross-stream direction and for the first streamwise modes.

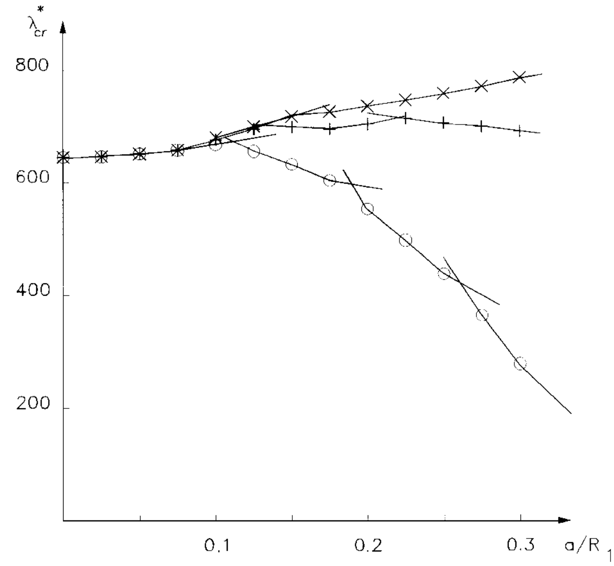

Fig. 1 Nondimensional aerodynamic critical pressure parameter $\lambda^*_{cr} = \lambda_{cr}a^3/E_2h^3$ vs variation of cross-stream curvature parameter a/R_1 for spherical shells (○), paraboloidal shells (+), and cylindrical shells (×) of fiber-reinforced laminated composite material having square planform and freely supported boundary conditions on the four edges.

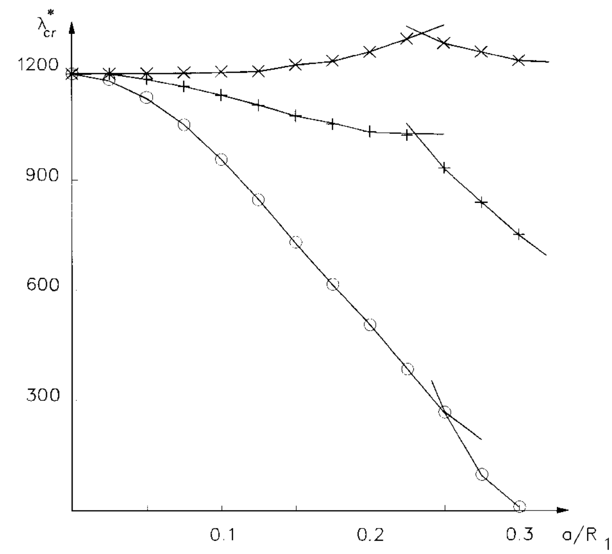

Fig. 2 Nondimensional aerodynamic critical pressure parameter $\lambda^*_{cr} = \lambda_{cr}a^3/E_2h^3$ vs variation of cross-stream curvature parameter a/R_1 for spherical shells (○), paraboloidal shells (+), and cylindrical shells (×) of fiber-reinforced laminated composite material having square planform and clamped boundary conditions on the four edges.

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